ALEKSANDR SAVVICH PREDVODITELEV

(To his 80th Birthday)

THE 11th of September 1971 was the 80th birthday of Professor Alaksandr Savvich Predvoditelev, Doctor of Physical and Mathematical Sciences, Corresponding Member of the USSR Academy of Sciences, State Prize Winner, Head of Molecular Physics Section at the Lomonosov Moscow State University.

Aleksandr Savvich Predvoditelev was born in 1891 in a poor peasant family in the village of Bukrin, Pronsk District, Ryazan Province.

Before he joined the army Predvoditelev's father was a poor farmer, sharing a small land plot with another male person of his family. During his military service he was advanced to the position of an orderly. After his military service he worked as a surgeon's assistant in various towns of Russia. He died of tuberculosis in 1907 at the age of 34. After her husband's death Aleksandr's mother worked at a factory but her scanty earnings could not provide for normal life of the family.

Aleksandr received his primary education in a village people school, then attended the Pronsk secondary school. When his parents moved to Ryazan, he entered the Ryazan town secondary school and upon finishing it, he was admitted to the grammar school (gymnasium). Predvoditelev's years in the gymnasium were full of material hardships and, after his father's death, his tuition fee was paid for him by the Ryazan Society to Assist Poor Gymnasium Students. In the upper classes he had to earn his living, working as a tutor in rich families.

After successfully finishing the gymnasium, Aleksandr entered the first course of the Physical and Mathematical Department of Moscow University. In spite of poor financial position Aleksandr took an active part in scientific research even in his first years at the University. He also spent a lot of time and efforts to extend his scientific knowledge. On graduating from the University, Predvoditelev, as one of the best students, was left at the Department to prepare to work as a professor.

From 1915 to 1919 Aleksandr Savvich worked as a Junior Assistant for practical work in physics at Moscow University, at the same time he carried out scientific research on optics and molecular physics.

In 1919 Aleksandr Savvich passed his master's (Magister) examination and, after delivering two test lectures, obtained the right to teach at the Physical and Mathematical Department of the Moscow University. From then on all his subsequent life has been connected with the University.

After 1919 he worked as a Senior Assistant for practical work in physics and delivered a number of special courses mainly in some branches of optics and molecular physics. Beginning from 1928 the authorities of the University of the Physical and Mathematical Department promoted A. S. Predvoditelev, first, to Vice-Chairman of the Subject Commission, then to Assistant of the Dean of the Physical and Mathematical Department and, finally, to the director of the Physical Research Institute of Moscow University where he remained till 1946.

In 1929 A. S. Predvoditelev began to treat Thermal Physics as a special field of physics, in 1930 he was appointed as Professor and Head of the Chair of Thermal Physics which is now called the Chair of Molecular Physics. He has been the head of this chair up till now.

Being the Dean of the Physics Department at

ALEKSANDR SAVVICH PREDVODITELEV

Moscow University A. S. Predvoditelev carried out extensive administrative and public activity from 1937 to 1946. Those were the most difficult years in the history of Moscow University. The work of the whole body of the university had to be rearranged for the needs of strengthening the defence power of our country.

From 1920 to 1923 A. S. Predvoditelev combined his work at the University with teaching physics at the Bauman Higher Technical School where he delivered lectures and guided practical work. From 1919 to 1930 he was a senior physicist (now senior scientific worker) at the Research Institute of Physics and Biophysics. From 1928 to 1932 A. S. Predvoditelev worked at the physical laboratory of the All-Union Institute of Labour Protection, and in 1929–1937 as a tutor and then as a scientific director at the Physical and Engineering Laboratory of the Dzerzhinsky All-Union Institute of Thermal Engineering. In 1938 A. S. Predvoditelev was invited to work at the Power Engineering Institute of the USSR Academy of Sciences, where he arranged and headed the Problem Laboratory of Combustion Physics up to 1967.

At all of the above research institutions A. S. Predvoditelev organized either large research laboratories or groups of researchers on various problems of modern physics.

A. S. Predvoditelev is a prominent Soviet scientist whose activity is not restricted to a narrow sphere of a single scientific or engineering problem. A wide range of scientific interests, high erudition, many-sided public activity and devotion to high ideals are characteristic of his life. The name of A. S. Predvoditelev is widely known both in our country and abroad, and a number of his brilliant and profound investigations in various fields of physical science have made a valuable contribution to the treasure of world science.

Predvoditelev obtained his most fundamental results in the solution of problems on gas and liquid state hydrodynamics, physics of combustion and explosion, physics of solids, physical acoustics of small and finite amplitudes, physical chemistry and physics of discontinuous processes.

The results of his experimental and theoretical studies are published in 200 monographs and papers.

A. S. Predvoditelev is a founder and organizer of the scientific school that is active in many research and educational institutions of this country.

Two features are peculiar to Predvoditelev's school. First, all the problems put forward by the interests of developing national industry and technology are physically based. Their practical study and solution are combined with theoretical analysis of the physical fundamentals of every problem considered. Investigations of physical and chemical processes in heterogeneous combustion carried out by Predvoditelev and his pupils, awarded with the State Prize in 1950, may serve as an example. Later on many scientists from Predvoditelev's school founded new independent bodies, groups, trends. Their activity has been progressing under the influence of the ideas developed at the laboratories directed by A. S. Predvoditelev.

Predvoditelev is a talented tutor of scientific workers who always encourages his coworkers to develop independent scientific views and approaches to the problems studied. Broad variety of his scientific ideas, high requirements to his work and the work of his assistants, ability to profound scientific analysis and high general erudition have been favouring the progresses of research groups directed by Predvoditelev and his numerous pupils.

A. S. Predvoditelev has trained more than 120 scientific researchers. Some 30 of this number are doctors and professors, two are corresponding members of the Academy of Pedagogical Sciences of the USSR, one is an Academician of the Byelorussian Academy of Sciences, one is a Corresponding Member of the Ukranian Academy.

A. S. Predvoditelev has been paying serious attention to the problems of history and methodology of physics. He has published

numerous essays and papers concerning the role of Russian scientists in development of fundamental ideas of modern physics. He was the initiator of publication of "Essays on History of Physics in Russia" (1949) and contributed some papers to the book. He is the Editor of two volumes of "Development of Physics in Russia" published in 1970.

A. S. Predvoditelev has greatly contributed to publication of fundamental works on history of physics by P. S. Kudryavtsev, B. I. Spassky and others. He has taken great efforts to acquaint the world scientific community with the achievements of Russian and Soviet scientists.

This is a general description of the life and work of A. S. Predvoditelev who has walked the way from a poor peasant to a prominent physicist.

The Government has highly estimated his scientific, pedagogical and organizing activity and awarded him the Lenin Order, four Orders of the Red Banner of Labour, the Order of Red Star and numerous medals.

LIST OF THE MAIN SCIENTIFIC WORKS BY A. S. PREDVODITELEV

1918
1. Light dissipation in oblique media, *Izv. Inst. Biofiz.*
2. L'indice d'absorption des millieux troubses. *Arch. Phys. Sc.*

1920
3. Necessary physical assumptions forming the basis for derivation of the radiation law, *Usp. Fiz. Nauk* II, vyp. I.
4. Reaction rate kinetic theory, *Usp. Fiz. Nauk* 2, vyp. I.
5. On the theory of gas reactions, *Usp. Fiz. Nauk* 2. vyp. I.
6. On thermal expansion of crystal solids, *Usp. Fiz. Nauk* 2, vyp. I.
7. Electronic theory of metals, *Usp. Fiz. Nauk* 2, 1.
8. Electric field and gas pressure effects on spectral lines. *Usp. Fiz. Nauk* 2, 1.
9. New theory of optical series, *Usp. Fiz. Nauk* 2. 1.

1923
10. One remarkable quantization case, *Usp. Fiz. Nauk* 3. 1.
11. Ammonia synthesis by the points of slow electrons, *Usp. Fiz. Nauk* 3. 1.

1925
12. Über der Wirklung der Schichtdicke auf den Photo-elektrischen Effekt in Farbstoffen, *Z. Phys.* 31, 7/8.
13. Absorption des Lichtes durch Ammoniak, *Z. Phys.* 6.
14. Zur Kinetik Photochemischer Prozesse in Farbstoffen. *Z. Phys.* 32, 3.
15. Zur Theorie des Abklingens der Fluoreszenz, *Z. Phys.* 32, 11/12.
16. Über die Spezifische Wärme des Wasserstoffmolekül, *Z. Phys.* 34, 2/3.
17. Über die Abhängigkeit des Absorptionskoeffizienten von der Dicke der Absorbierten Farbschicht. *Z. Phys.* 35. 1.

1926
18. Zur Frage nach der Photoelektrischen Ermildung, *Z. Phys.* 35, 10.
19. Über die Abhängigkeit der Flüssigkeitsdichten von der Temperatur, *Z. Phys.* 36, 7.
20. Der Einfluss des Absorbierten Gases auf die Grösse des Photoelektrischen Effektes, *Z. Phys.* 38, 4/5.
21. Zur Frage der Abhängigkeit der Dichte von der Temperatur, *Z. Phys.* 40, 6.

1927
22. Über den Einfluss des Kristallwassers auf den Photo-elektrischen Effekt in den Kristallhydraten (Teil I), *Z. Phys.* 42, 1.
23. Über den Einfluss des Kristallwassers auf den Photo-elektrischen Effekt in den Kristallhydraten (Teil II). *Z. Phys.* 44, 3.
24. On chemical reaction kinetics of solid and gaseous components that result in formation of compounds. *Zh. Prikl. Fiz.* 4, 3.

1928
25. Über die Gesetze der Molekülstösse mit einer Ober-fläche mit welcher das Gas Reagiert, *Z. Phys.* 46, 5/6.
26. Das Maxwelliche Relaxationsgesetz und die Innere Reibung der Flüssigkeiten, *Z. Phys.* 49, 3/4.
27. One possible method of measuring heating of equipment for chemical compounds which may dissociate in vacuum, *Zh. Prikl. Fiz.* 5 (additional issue).
28. *What is Light Recording?* Nauka i Tekhn., Izd. Mosk. Rab.
29. Über die Absoluten Geschwindigkeiten der H_2O-Moleküle, welche bei der Dehydration von Kristall-hydraten Herausfliegen, *Z. Phys.* 51, 1/2.
30. Zur Kinetik Chemischer Reaktionen zwischen einer Festen und einer Gasförmigen Komponente, die zur Bildung Komplexer Verbindungen Führen, *Z. Phys., Chem.* 132, 1/2.

1929
31. Berichtigung zur Arbeit über die Absoluten Gesch-windigkeiten der H_2O-Moleküle, welche bei der De-hydration von Kristallhydraten Herausfliegen. *Z. Phys., Chem. Bd.* 54, 1/2.
32. One geometric interpretation of hydrogen atom structure following Rutherford–Bohr, *Zh.R.F.Kh.O.* 61, 5.

33. On possibility of relation between Bohr postulates and de Broglie–Schroedinger equation, *Zh.P.F.Kh.O.* **61**, 5.
34. On measuring heat of dehydration in vacuum by torsion balance (with co-author L. SELIVANOV), *Zh. Prikl. Fiz.* **5**, 3–4.

1933
35. On the theory of fluids and viscosity of associated molecules, *Zh. Eksp. Teor. Fiz.* **3**, 3.
36. Viscosity of liquids and gases from the viewpoint of cyclic motion, *Zh. Eksp. Teor. Fiz.* **3**, 3.

1934
37. Stability of mechanical motions in atomic physics, *Zh. Eksp. Teor. Fiz.* **4**, 1.
38. Heat of strong electrolytes, *Zh. Eksp. Teor. Fiz.* **4**, 1.
39. Relation between thermal conductivity, heat capacity and viscosity of liquid bodies, *Zh. Eksp. Teor. Fiz.* **4**, 1.
40. Thermal conductivity of solid heat insulators, *Zh. Eksp. Teor. Fiz.* **4**, 8.
41. Thermal conductivity of electroconducting bodies, *Zh. Eksp. Teor. Fiz.* **4**, 8.

1935
42. A contribution to the theory of the Bunsen flame, *Techn. Phys. USSR* **11**, 4.
43. A contribution to the problem of the decay of a gas jet discharging from a pipe, *Techn. Phys. USSR* **11**, 4.
44. Theory of gas reactions in high-frequency electric discharge, *Zh. Fiz. Khim.* **6**, 4.
45. Jet attenuation at gas flow out of tube (with co-author E. V. STUPOCHENKO), *Zh. Tekh. Fiz.* **5**, 8.
46. On the Bunsen flame theory, *Zh. Tekh. Fiz.* **5**, 8.
47. Solid admixture effect on flame propagation velocity in fuel gas mixtures (with co-author S. I. GRIBKOVA), *Zh. Tekh. Fiz.* **7**, 18–19.

1940
48. Burning of coal channel walls at forced oxygen diffusion (with co-author O. A. PUKHANOVA), *Zh. Tekh. Fiz.* **10**, 13.
49. Burning of coal particle in a gas flow, *Zh. Tekh. Fiz.* **10**, 16.

1941
50. Theory of coal channel burn-out, *Zh. Tekh. Fiz.* **11**, 10.

1945
51. Experience of constructing kinetics of sulphur dioxide oxidation in electric discharge, *Uch. Zap. MGU* **75**, kn. 2, ch. 2.

1946
52. Physical problems of heat engineering, Izv. Vses. Tepl. Inst. 9–10.

1947
53. Theory of heterogeneous combustion and mechanism of solid fuel burning in a bed (with co-author Kh. I. KOLODTSEV). Proc. World Power Engng Conference, Hague Econom. Congr., Gosenergoizd., Moscow.

54. Possible classification of statistical systems, *Vestnik MGU*, 7.
55. The Arrhenius law in chemical kinetics of gas reactions, *Dokl. Akad. Nauk SSSR, Otd. Khim.*, 57, 7.
56. Gasification at coal burning in a bed, *Izv. Akad. Nauk SSSR*, ONTI, 10.
57. Quasi-steady state of chemical gas systems, *Vestnik MGU*, 10.

1948
58. Fluctuations in statistical systems, *Vestnik MGU*, 4.
59. Theory of acoustic dispersion, *Vestnik MGU*, 5.
60. On statistical assembly of material points of variable mass, *Vestnik MGU*, 6.
61. On molecular-kinetic foundations of hydrodynamic equations, *Izv. Akad. Nauk SSSR, ONTI*, 4.
62. Some invariant quantities in the theory of thermal conductivity and viscosity of fluids, *Zh. Fiz. Khim.* **22**, 3.

1949
63. *Carbon Burning* (with co-authors L. N. KHITRIN, O. A. TSUKHANOV, KH. I. KOLODTSEV and M. K. GRODZOVSKY). Izd. Akad. Nauk SSSR, Moscow–Leningrad.
64. Some considerations on the operator method in wave mechanics, *Vestnik MGU*, 2.
65. On thermal motion in condensed media and their state equations, *Vestnik MGU*, 3.
66. Modification of Zhukovsky's equation for disturbed motions of statistical systems, *Vestnik MGU*, 5.
67. Some applications of Zhukovsky's equation for disturbed motions of statistical systems, *Vestnik MGU*, 6.
68. Some remarks on nature of elementary particles, *Vestnik MGU*, 8.

1950
69. On molecular heat transfer in liquids, *Dokl. Akad. Nauk, Otd. Fiz. Khim.* **72**, 2.
70. Clausius theorem of mean ergal and stable statistical systems, *Vestnik MGU, Otd. Fiz.*, 6.

1953
71. The state of art in the theory of homogeneous–heterogeneous combustion, *Izv. Akad. Nauk SSSR, Otd. Tekh. Nauk*, 5.

1956
72. *Thermal Conductivity and Viscosity of Liquids and Compressed Gases.* Collected Papers, volume dedicated to P. P. LAZAREV, Izd. Akad. Nauk SSSR, Moscow.

1957
73. *Tables of Thermodynamic Functions of Air* (with co-authors E. V. STUPOCHENKO, I. P. STAKHANOV, E. V. SAMUILOV, A. S. PLESHANOV and I. B. ROZHDESTVENSKY). Izd. Akad. Nauk SSSR, Moscow.
74. General properties of discontinuous processes and acoustic effects in nonhomogeneous solids, in *Application of Ultra-acoustics to Substance Investigation.* Izd. MOPI, vyp. 4.

1958

75. One method of acoustic effect description, *Application of Ultraacoustics to Substance Investigation.* Izd. MOPI, vyp. 6.
76. On the so-called Rao rule, *ibid.*, 7.
77. On the possible method of describing structural non-uniformities in liquids, *ibid.*
78. Theoretical examination of vibratory movement of the flame front in closed vessel, Trans. 7th International Symposium on Combustion and Explosion.
79. On automodelling processes in chemically active media, *ibid.*
80. On spin detonation, *ibid.*

1959

81. On the theory of adsorption wave. *Dokl. Akad. Nauk. Otd. Fiz. Khim* **127**, No. 3; 127, No. 4.
82. *Movement of Combustion zone as Hydrodynamic Inhomogeneity.* Collected Papers, Volume Dedicated to G. M. KRZHIZHANOVSKY.
83. Hydrodynamic inhomogeneities in combustion and explosion theory, *Gasdynamics and Combustion Physics.* Izd. Akad. Nauk SSSR, Moscow.
84. Dispersion of acoustic waves in rarefied gases (Paper 1), *Application of Ultraacoustics to Substance Investigation.* Izd. MOPI, vyp. VIII.

1960

85. Dispersion of acoustic waves in rarefied gases (Paper 2), *ibid.*
86. Combustion of spherical particles, Proc. Higher School Conf. on Combustion, Izd. Odess. Univ. 150, 6.
87. Front evaporation and sublimation of spherical particles, *ibid.* 150, 7.
88. Velocity of combustion front propagation in two-phase mixtures, *ibid.*, 150, 7.

1961

89. On chemical reaction rates in turbulent flows. *Inzh. Fiz. Zh.* **4**, 1.
90. On thermal motion type in liquids, *Inzh. Fiz. Zh.* **4**, 6; 7; 8.
91. On transverse and longitudinal wave dispersion in relaxation media, Vestnik Mosk. Univ., ser. III, No. 3.
92. On acoustic wave dispersion in relaxation media, Vestnik Mosk. Gos. Univ., ser III, No. 4.
93. On aerodynamics of rarefied gases and problems of heat transfer, *Int. J. Heat Mass Transfer* **3**.

94. On condensed media state equation (Theoretical Portion), *Inzh. Fiz. Zh.* **5**, No. 8.
95. On condensed media state equation (Experimental Portion), *Inzh. Fiz. Zh. Zh.* **5**, No. 11.
96. On conservation laws at the shock-wave front, *Physical Gas Dynamics,* Heat Transfer and Gas Thermodynamics at High Temperatures, Izd. Akad. Nauk SSSR, Moscow.
97. On one inconsistency of the Zjuge Theory on regular motion of shock breaks with spherical and cylindrical symmetry, *ibid.*
98. On molecular-kinetic consideration of the Chapman–Zjuge first rule, *ibid.*

1963

99. On critical phenomena of liquids and gases, *Inzh. Fiz. Zh.* **VI**, 4, 123.
100. The teaching of space and time, modern science. *History and Methodology of Natural Sciences.* Izd. MGU.
101. On helium state equations, *Inzh. Fiz. Zh.* **VI**, No. 6, 54.
102. On kinetics of statistical motion, *Inzh. Fiz. Zh.* **VI**, No. 9, 87.
103. On state equations of carbon monoxide and dioxide, *Inzh. Fiz. Zh.* **VI**, No. 12, 100.

1964

104. On state equations of xenone and methane, *Inzh. Fiz. Zh.* **VII**, No. 1, 93.
105. On the general theory of wave processes, Collected Papers, *Physical Gas Dynamics and Gas Properties at High Temperatures.* Izd. Akad. Nauk SSSR.

1965

106. Mathematical computation and our knowledge. *History and Methodology of Natural Sciences.* Izd. MGU.

1967

107. To the theory of two-phase mixtures, Collected papers, *Gas Properties at High Temperatures,* ENIN.
108. On conditions of ignition in fuel gas mixtures, *ibid.*

1970

109. Human Being and Reality, *History and Methodology of Natural Sciences.* Izd. MGU.
110. On physical basis of hyperbolic differential equations, Uch. Zap. Krasnodarsk. Politechn. Inst.
111. Theory of heat and Riemannian manifolds. Collected Papers, *Heat and Mass Transfer Problems,* Izd. Energia.

ON A. S. PREDVODITELEV'S INVESTIGATIONS INTO HYDRODYNAMICS AND HEAT TRANSFER

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Hydrodynamic motions represent a wide range of natural processes. Therefore, hydrodynamic methods which allow transition from microscopic description of the natural processes to macroscopic one are widely used in various fields of physics.

Different degree of accuracy of the description of the above transition corresponds to different forms of hydrodynamic continuum equations. The ideal liquid model described by hydrodynamic equations in the Euler form proved to be valid for a narrow range of the natural processes. Presumably, the incompleteness of these equations was originally expressed in the d'Alembert paradox.

The discovery of internal friction in liquids and gases allowed the hydrodynamic equations in the Euler form to be supplemented and specified. For this we are indebted to Navier, Poisson, Saint-Venant and Stokes. The first of the above scientists derived equations for viscous fluid from the molecular-kinetic theory based on various considerations of molecular interaction. However, Navier did it as far back as 1822, while Poisson seven years later. Alternative method for deriving the above equations was originally suggested by Saint-Venant in 1843 and Stokes in 1845. Now this method is referred to as phenomenological.

Stokes has succeeded to prove that the conditions of adhesion exist at the liquid–solid wall boundary and thus he was first to formulate the boundary conditions and put an end to the argument lasting from Euler's and Bernoulli's times.

At present the hydrodynamic equations of this form are named the Navier–Stokes equations and constitute the basis of classical hydrodynamics.

In 1866 the memoirs *On Dynamic Gas Theory* by D. K. Maxwell appeared concerning the method which may be used to obtain all heat and mass transfer equations.

In this paper Maxwell discovered the molecular-kinetic structure of Navier–Stokes equations, in particular, he has shown that in the Navier–Stokes continuum the visible (macroscopic) and thermal (microscopic) motions coexist.

Hagen and Osborne Reynolds were the first who revealed incompleteness of the Navier–Stokes equations while studying water motion along the tubes of various diameters. Reynolds has shown that at certain conditions defined by a dimensionless number later named after him, the flow pattern begins changing sharply and experimental data no longer agree with Poiseuille solution. Later this phenomenon was referred to as turbulence and defined by some critical Reynolds number.

For a long time the hydrodynamic Navier–Stokes equations did not rise any doubts under the so-called laminar flow conditions below the critical Reynolds numbers. This was partly accounted for by huge mathematical difficulties involved in general solutions of the above equations.

However, reported developments in mathematical physics and aircraft engineering and new materials revealed inadequacy of the Navier–Stokes equations even in the laminar flow region. The necessity arose to refine the above equations. This problem was most sharply stated by the American scientist Truesdell in "Modern Crisis in the Kinetic Gas Theory", 1958.

Professor Predvoditelev's hydrodynamic investigations are connected with this problem.

1. *Fundamental equation of kinetic theory*

Nowadays the hydrodynamic equations are derived from solutions of the Boltzmann integro-differential equation by the Chapman–Enskog method. The idea of this method was known to Boltzmann; it was presented in his lectures on the gas theory. However, in the greater portion of the book studying transfer processes Boltzmann deals with Maxwell's method.

He wrote:*

"Up till now we followed the brilliant method of Maxwell which has been also used by Kirchhoff and others. The essence of this method is that it not at all connected with calculation of the function $f(x, y, z, \xi, \eta, \zeta, t)$ defining the velocity distribution. There exists an alternative method: it just proceeds from calculation of this function. Though this method did not attract any attention, nevertheless, I would like to touch upon it in some words as we shall need this very function f when calculating entropy".

In his hydrodynamic investigations A. S. Predvoditelev has never used the Boltzmann equation. He expressed his view on it in the paper submitted to the 2nd All-Union Heat and Mass Transfer Conference, Minsk, 1964. By that time a number of experimental data had been accumulated which came in conflict with the solutions of Boltzmann's integro-differential equation.

In this paper Predvoditelev supposed that if the methods of solving the Boltzmann equation are quite perfectly right then the contradictions spoken of may be attributed to incompleteness of this equation. Further, he analyzed physical significance of the Boltzmann equation as follows:

"As is known, two operations at the molecule distribution function with respect to heat velocities (f) are equated. The first operation is used to calculate change of the function with respect

* L. Boltzmann, *Lectures on Gas Theory*, Translation from German, ed. B. I. Davydov. Gostekhizdat, Moscow, pp. 225 (1956).

to time, coordinates and velocities per unit time. This change may be written down as follows:

$$\frac{Df}{Dt} = \frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + v\frac{\partial f}{\partial y} + w\frac{\partial f}{\partial u} + X\frac{\partial f}{\partial v} + Y\frac{\partial f}{\partial v} + Z\frac{\partial f}{\partial w}$$

here the equalities are implied:

$$\frac{\partial u}{\partial t} = X, \frac{\partial v}{\partial t} = Y, \frac{\partial w}{\partial t} = Z.$$

The second operation is used to calculate the change (per time unit) of the same function due to molecular collision. If velocity components of the first type of molecules are designated by ξ_1, η_1, ζ_1 and those of the second type of molecules before collision by ξ_2, η_2, ζ_2, then the probability measure for their approaching each other may be taken equal to $f_1 f_2$. Subscripts indicate the components of heat velocities of which the distribution function must be taken. Upon collision, velocity components of the first and second type will take the values

$$(\xi_1', \eta_1', \zeta_1'), (\xi_2', \eta_2', \zeta_2')$$

and the product $f_1' f_2'$ will serve as a probability measure of increasing distances between them. Thus each collision of one molecular group with another is characterized by the difference

$$f_1' f_2' - f_1 f_2 = \delta f_1 f_2$$

Summation of these differences over the whole phase volume gives the change of f function per time unit due to molecular collision. Thus we have

$$\frac{Df}{Dt} = \quad \dots \quad \delta f_1 f_2 Vb \, db \, d\omega_1 \, d\phi.$$

Here the product $Vb \, db \, d\omega_1 \, d\phi$ denotes a unit phase volume where V is the relative velocity of colliding molecules; b is the shortest distance between straight-line trajectories of molecules; ϕ is the angle between the plane of trajectories of relative motion of the first type of

molecules and some uniform plane crossing V: $d\omega_1$ is the product $d\xi_1 \, d\eta_1 \, d\zeta_1$. The question arises whether the operations discussed are equal. The first operation may give continuous values of the phase space coordinates. The same is not evident for the second operation. I am inclined to believe that the second operation should lead to discontinuous values, as the variation $\delta f_1 f_2$ may be not only infinitesimal. This implies that equating the operations discussed to each other is principally wrong. As is known, these are equallized without thinking over, i.e. it is taken for granted that the variation $\delta f_1 f_2$ is a continuous function of the phase space coordinates. I believe that validity of such equalizing will increase with the density of gas".

The criticism however was not substantiated enough. At this time Predvoditelev completed the generalization of the Navier–Stokes equations and these generalized equations did not follow from the solutions of Boltzmann's equation. Moreover, in this paper experimental check of these equations was made in the case of acoustic wave dispersion. In particular, the following formula was obtained for the sound absorption coefficient in a rarefied gas based on Predvoditelev's equations discussed

$$\alpha = \frac{(0\cdot7r + 0\cdot23)\sqrt{r}}{(r + 0\cdot575)^2 \sqrt{(r + 0\cdot575)}}.$$

Here r is the known dimensionless quantity in acoustics which is the ratio of pressure to the product of viscosity by frequency.

Further, it appeared that Predvoditelev's formula agrees better with Greenspan's observations of acoustic wave dispersion in xenon and krypton compared to other available formulae.

Returning again to Boltzmann's collision integral, it should be noted that discontinuity of the variation $\delta f_1 f_2$ emphasized by Predvoditelev is caused by discontinuous behaviour of the distribution function at the moment of molecular collision.

Boltzmann realized that and in his last years in the discussion of F. Klein and A. Hefler's

paper at the Colloquium of the Vienna Philosophical Society he expressed very curious ideas concerning statistical description of physical effects. He considered that in a statistical system of material points the trajectory of position of the points and trajectories of their velocity are continuous functions that had no derivatives at any point. This means that for a given trajectory of position, calculation of velocity is impossible, and positions of a material point of a statistical system cannot be determined from the given trajectory. Thus, for any physical system we cannot but have two sets: a set of positions and a set of velocities. Moreover, we should choose such a calculation method that would allow analytical relationships to be found which would not violate the above principle of Boltzmann.

In his recent work "The Science of Heat and the Riemannian Manifolds" Predvoditelev further developed the idea of the velocity distribution function as continuous and differentiable over the free molecular path length at turning points of their trajectories. First of all, in this work he developed the following representation of a statistical system:

"We shall concentrate on a certain molecule of gas. In space we shall draw sections of the molecule path between two collisions within a very long time. As a result our limited space will be filled almost completely with molecule trajectory. Also turning points of the molecule after its collision with others will fill the space almost completely. If we draw a curve over these points, it will form a tangle. The curve will be continuous and nondifferentiable and will almost completely coincide with the trajectory of molecule position plotted. This tangled curve will be a locus of molecular velocity deviations from a mean velocity. Density of points on this line is nonuniform. A certain heat velocity of a molecule with the same direction as that of the trajectory will correspond to each point."

Here Predvoditelev is trying to prove that this physical image of a statistical system does not contradict Maxwell's conception. With this

aim we shall take some continuous and differentiable function with heat velocity of a molecule as an argument. We assume that for each point of the curve drawn through the turning points, a single-valued correspondence of the values of the above function may be found, the function being denoted by f.

As the turning point is the place of collision of two molecules, another trajectory passes through it which corresponds to another colliding molecule. The distribution function for heat velocities of the points of this trajectory is designated by F.

If we denote the value of the function f after turn of the first molecule trajectory by f_1, then the function jump is

$$f_1 - f.$$

According to Dirichlet, the value of the same function at the break point may be assumed equal to

$$\frac{f_1 + f}{2}.$$

If gas is in a steady state, then the relation holds

$$\frac{2(f_1 - f)}{(f_1 + f)} = \text{const.}$$

In a similar way it is possible to find the relation for the function F also:

$$-\frac{2(F_1 - F)}{(F_1 + F)} = \text{const.}$$

Here we see that the minus sign appears since increase of the function f corresponds to decrease of the function F.

Further we can write

$$\frac{f_1 - f}{f_1 + f} = \frac{F - F_1}{F_1 + F}. \tag{1.1}$$

From this equality the famous functional Maxwell equation may be easily obtained

$$fF = f_1 F_1. \tag{1.2}$$

Predvoditelev, however, did not restrict himself by the Maxwell equation (1.2). In the said work he tries to obtain a special method for description of statistical effects without the concept of velocity distribution function. With this aim, we preliminarily suppose that cross point of intersection of colliding molecules forms a continuous surface in the space, no property of the system being possible to change continuously in transition through this surface.

Existence of discontinuity surfaces should not be necessarily understood as collision of molecules. It may be supposed that in these sites sharp jumpwise change of some properties of the substance occurs and the cause of such changes may be considered the behaviour of thermal motion in the substance.

The said discontinuous surface is denoted by S. Let partial derivative of this surface with respect to time be equal to the total energy with the reverse sign; partial derivatives with respect to the coordinates are equal to the corresponding momenta. These properties of the surface allow the Helmholtz equation to be written for it

$$\frac{\partial S}{\partial t} + \frac{1}{2m} \sum \frac{\partial S}{\partial x_i}^2 = U. \tag{1.3}$$

This equation implies that molecular collision occurs following the energy conservation law.

The function which determines the state of a statistical system before and after collision is denoted by $\Psi(x_1, x_2, x_3, t)$. Let it possess the following properties: the first derivatives with respect to coordinates are equal to momenta; the first derivative with respect to time is equal to a certain value that has the energy dimension. Such a function of the state should undergo discontinuities of all the first and second derivatives in transition through the surface S. Besides, the very function that in a sense determines the structure of the assembly should also undergo discontinuity as molecular collisions should be accompanied by disturbance of structural properties of statistical assembly as well.

Generalizing the conditions of commensurability of Hugoniot–Hadamard for the cases when discontinuities are observed that change in time and space, Predvoditelev showed that for the case considered the following commensurability conditions should be valid

$$\left.\begin{aligned}
\frac{H}{\lambda_1}\delta\frac{\partial\Psi}{\partial t} &= \frac{\partial S}{\partial t}, \frac{H}{\lambda_1}\delta\frac{\partial\Psi}{\partial x_i} = \frac{\partial S}{\partial x_i}, \\
\frac{H^2}{\lambda^2}\delta\frac{\partial^2\Psi}{\partial x_i^2} &= \left(\frac{\partial S}{\partial x_i}\right)^2, \lambda_1^2 = \lambda_2\lambda_0, \frac{\lambda_0}{\lambda_1}H = A, \\
\frac{\lambda_1}{\lambda_2}H &= A.
\end{aligned}\right\}(1.4)$$

Here H is the first differential invariant of the function S; $\lambda_0, \lambda_1, \lambda^2$ are arbitrary factors referred to as discontinuity parameters of the corresponding order.

Substitution of equation (1.4) into equation (1.3) yields

$$\frac{H}{\lambda_1}\delta\frac{\partial\Psi}{\partial t} + \frac{1}{2m}\frac{H^2}{\lambda_2}\delta\Delta_2\Psi = U.$$

With assumption $\lambda_0 = \delta\Psi$, the above equality is symmetrical with respect to operation δ:

$$\frac{\lambda_0}{\lambda_1}H\delta\frac{\partial\Psi}{\partial t} + \frac{1}{2m}\frac{\lambda_0 H^2}{\lambda_2}\delta\Delta_2\Psi = U\delta\Psi$$

or

$$A\delta\frac{\partial\Psi}{\partial t} + \frac{A^2}{2m}\delta\Delta_2\Psi = U\delta\Psi. \qquad (1.5)$$

In the equality obtained all factors at operation δ are continuous. Therefore, a reverse operation may be applied to equality (1.5). Then

$$A\frac{\partial\Psi}{\partial t} + \frac{A^2}{2m}\Delta_2\Psi = U\Psi + F(x_1, x_2, x_3, t).$$

Here arbitrary and continuous function $F(x_i, t)$ may be assumed equal to zero. Then

$$A\frac{\partial\Psi}{\partial t} + \frac{A^2}{2m}\Delta_2\Psi = U\Psi. \qquad (1.6)$$

Equation (1.6) is the same as the Schrödinger

equation. It becomes completely identical with the latter if we assume

$$A = \frac{h}{2\pi i}$$

Thus, Predvoditelev has reduced searching for the distribution function of a statistical system to searching for the integral in equation (1.6).

Predvoditelev's method just discussed reveals quite a new approach to searching for velocity distribution functions and functions that determine structural peculiarities of statistical systems.

2. On Navier–Stokes equation generalization

N. P. Kasterin's research on vortex motions became a starting point for Predvoditelev in his attempt to generalize Navier–Stokes equationr.

Kasterin was the first in the Soviet Union who discussed the necessity to revise Euler equations with reference to vortex flows. He was probably the first who discovered that vortex motion initiation is connected with discontinuities appearing in hydrodynamic velocity field.

However, in derivation of the Euler equations smooth change of velocities within physically small volume is postulated. That is the reason why the Helmholtz theorems based on the Euler equations disagree with experimental observations of vortices.

This view of Kastering contradicted the generally adopted opinion that vortices arising may be attributed to viscous force effect.

In 1937 Kasterin made an attempt to include discontinuous change of velocities in hydrodynamic equations for the ideal liquid. For example, for potential flows Kasterin's equation is of the form

$$\frac{\partial V}{\partial t} - \text{grad}\frac{V^2}{2} = -\frac{1}{\rho}\text{grad }p. \qquad (2.1)$$

The only difference from the Euler equation is a different sign before the gradient operation in the left-hand side of equation (2.1).

In June 1947 in his joint researches with A. K. Timiryasev and T. M. Sviridov on condi-

tions for sandstorms to arise in the laboratory, Kasterin experimentally confirmed the said change of the sign.

A. S. Predvoditelev decided to create a molecular-kinetic basis for the sign change. With this aim he made use of Maxwell's method.

If Q is the qualitative expression of some of the properties transferred by moving molecules, then Maxwell's equation assumes the form

$$\frac{\partial \overline{QN}}{\partial t} + \overline{QN}\left(\frac{du}{dx} + \frac{dv}{dy} + \frac{dw}{dz}\right) + \frac{d}{dx}(\overline{\xi QN})$$

$$+ \frac{d}{dy}(\overline{\eta QN}) + \frac{d}{dz}\,\overline{\zeta QN}) = \frac{\delta Q}{\delta t}\,N. \qquad (2.2)$$

Here

$$\frac{\delta Q}{\delta t}\,N$$

is the collision integral

$$\frac{\delta Q}{\delta t}\,N = (Q' - Q)\,Vb\,db\,d\phi f_1 f_2\,d\omega_1\,d\omega_2.$$

$$(2.3)$$

The remaining notations are those generally adopted.

However, to obtain hydrodynamic Navier–Stokes equations from equation (2.2) Maxwell used two hypotheses.

Hypothesis A. Molecular motion is caused by central forces. Molecular interaction force is a repulsive force which is inversely proportional to the fifth power of the distance between interacting molecules.

Hypothesis B. Transfer rates of two colliding molecules are equal within a physically small volume over which averaging is carried out.

Maxwell applied the above interaction law to solution of a number of molecular-kinetic problems, and in 1905 Langevin confirmed his calculations starting from any law of molecular interaction forces.

The second hypothesis of Maxwell is related to the problem on determination of hydrodynamic velocity in a moving gas. Within the domain of validity of the said hypothesis,

Maxwell identified visible velocity of moving gas with mean velocity of thermal molecular motion. Thus, he assumed that within the domain of validity of Navier–Stokes equations visible and thermal motions coexist and there is no gradient of hydrodynamic velocities within a physically small volume.

Such a flow structure is not always the case. At rather large velocities such a motion in continuum may be disturbed near the wall or when vortices are formed. This hypothesis becomes doubtful in the cases also when flow dimensions are of the same order as mean distances between molecules that occurs in motion of rarefied gases.

In 1948 Predvoditelev paid attention to this weak point of Maxwell's hypothesis and formulated some new conditions for transition to continuum.

If subscripts 1 and 2 indicate transfer rates of the first and second molecules, respectively, then following Maxwell, we obtain the following formula for the stress tensor in a very general case

$$\left.\begin{aligned} \rho\overline{\xi^2} &= p + \frac{A_1}{3A_2}\rho(u_2^2 - u_1^2) \\ &\qquad - 2\mu\left[\frac{\partial u}{\partial x} - \frac{(\gamma-1)}{2}\mathrm{div}V\right] \\ \rho\overline{\xi\eta} &= -\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \\ &\qquad + \frac{A_1}{3A_2}\rho(u_2 v_2 - u_1 v_1) \end{aligned}\right\}(2.4)$$

where constants A_1 and A_2 are defined by the interaction law.

Definition of hydrodynamic velocity must be refined. For this we assume that vector V with projections u, v, w is superimposed on a certain physically small volume with molecules involved in heat motion of which transfer velocities are different. Then projection of the material point with respect to a fixed coordinate system may be expressed as

$$V_x = u + \xi,\ V_y = v + \eta,\ V_z = w + \zeta. \quad (2.5)$$

Different transfer velocities of all the molecules within a physically small volume create hydrodynamic velocity gradient, therefore vector V may be treated as velocity of the gravitation centre of this volume.

Equations (2.4) and (2.5) allow the following set of equations to be obtained from equation (2.2):

$$\rho \frac{du}{dt} + \frac{A_1}{3A_2} \left\{ \frac{\partial}{\partial x} \ [\rho(u_2^2 - u_1^2)] \right.$$

$$+ \frac{\partial}{\partial y} \ [\rho(u_2 v_2 - u_1 v_1)]$$

$$\left. + \frac{\partial}{\partial z} \ [\rho(u_2 w_2 - u_1 w_1)] \right\} = - \frac{\partial p}{\partial x}$$

$$+ \mu \left[\nabla^2 u + (2 - \gamma) \frac{\partial}{\partial x} \operatorname{div} V \right],$$

$$\rho \frac{dv}{dt} + \frac{A_1}{3A_2} \left\{ \frac{\partial}{\partial x} \ [\rho(u_2 v_2 - u_1 v_1)] \right.$$

$$\left. + \frac{\partial}{\partial y} \ [\rho(v_2^2 - v_1^2)] + \frac{\partial}{\partial z} \ [\rho(v_2 w_2 - v_1 w_1)] \right\}$$

$$= - \frac{\partial p}{\partial y} + \mu \left[\nabla^2 v + (2 - \gamma) \frac{\partial}{\partial y} \operatorname{div} V \right],$$

$$\rho \frac{dw}{dt} + \frac{A_1}{3A_2} \left\{ \frac{\partial}{\partial x} \ [\rho(u_2 w_2 - u_1 w_1)] \right.$$

$$\left. + \frac{\partial}{\partial y} \ [\rho(v_2 w_2 - w_1 v_1)] + \frac{\partial}{\partial z} \ [\rho(w_2^2 - w_1^2)] \right\}$$

$$= - \frac{\partial p}{\partial z} + \mu \left[\nabla^2 w + (2 - \gamma) \frac{\partial}{\partial z} \operatorname{div} V \right]. \quad (2.6)$$

where γ is the index of Poisson adiabatic curve.

Conditions for transition to continuum presented by Predvoditelev may be reduced to the following hypothesis that expresses transfer velocities of colliding molecules in terms of hydrodynamic velocity vector

$$u_1 = u + A \left[(x - x_0) \frac{\partial u}{\partial x} + (y - y_0) \frac{\partial u}{\partial y} \right.$$

$$\left. + (z - z_0) \frac{\partial u}{\partial z} \right]$$

$$u_2 = u - A \left[(x - x_0) \frac{\partial u}{\partial x} + (y - y_0) \frac{\partial u}{\partial y} \right.$$

$$\left. + (z - z_0) \frac{\partial u}{\partial z} \right] \quad (2.7)$$

where x_0, y_0, z_0 are gravitation centre coordinates of a physically small volume. Similar formulae are implied for the components v_1, v_2, w_1, w_2.

Now instead of equation (2.6) we have Predvoditelev's equation

$$\frac{\partial V}{\partial t} + \tfrac{1}{2}(1 - \beta) \operatorname{grad} V^2 - \beta V \operatorname{div} V$$

$$+ (1 - \beta) \operatorname{rot} V \times V = - \frac{1}{\rho} \operatorname{grad} p + \nu \nabla^2 V$$

$$+ \nu(2 - \gamma) \operatorname{grad} \operatorname{div} V. \quad (2.8)$$

β designates the quantity $\beta = 2AA_1/3A_2$ that was later called by Predvoditelev a nonideal continuity parameter.

Predvoditelev's equation (2.8) is more general than the Navier–Stokes equation which Maxwell tried to justify in terms of molecular-kinetic theory.

In fact, if we assume $\beta = 0$ and $\gamma = \tfrac{5}{3}$ that corresponds to monoatomic gas, then equation (2.8) becomes identical to the Navier–Stokes equation. But this is not always the case.

Further, Maxwell did not notice that his limitation implicitly involved adiabatic equation for monoatomic gas, therefore instead of $(2 - \gamma)$ he obtained $\tfrac{1}{3}$.

Now it may be easily seen that Predvoditelev's equation becomes Kasterin's equations if $\beta = 2$, $\operatorname{div} V = 0$, $\nu = 0$.

In his investigations on acoustics Predvoditelev expressed nonideal continuity parameter in terms of Knudsen and Mach numbers

$$\beta = \tfrac{3}{2} Kn\, M \quad (2.9)$$

and demonstrated that equation (2.8) described well Greenspan's experiments in a wide range of Knudsen's number.

3. On turbulent flows

Hagen was probably the first who observed a special flow pattern that was later called turbulent flow by Lord Kelvin. Hagen found that up to a certain velocity limit liquid flow consists of jets which have the form of infinitely thin rods, the flow dimension being prescribed. After the limit is reached, motion becomes immediately nonuniform; jets disintegrate and begin to fluctuate. Hagen also paid attention to the fact that the transition from jet flow to turbulent flow depends on viscosity and dimensions of the flow. However, he did not discover general principles that may define commencement of transition.

The honour of this discovery belongs to Osborn Reynolds, Maxwell's pupil· who demonstrated that transition of one type of flow to the other is characterised by a special dimensionless criterion referred to as the Reynolds number. He reported this discovery in his work published in 1894 in which he also presented statistical method for writing the equation of turbulent flow.

Reynolds realized that the dynamic theory of Maxwell is based on geometrical division of the true molecular velocity into transfer and thermal ones, transfer velocity being mean velocity of thermal molecular motion which coincides with hydrodynamic velocity within the Navier–Stokes continuum.

Mean value of thermal velocity is zero according to Maxwell's conception.

This geometrical basis of Maxwell's method was adopted by Reynolds for physical representation of turbulent flow. For this he suggested to divide the velocity at any point, obtained from formulae similar to those of kinetic gas theory, into mean, referred to now as averaged velocity and relative velocity

$$u = \bar{u} + u', v = \bar{v} + v', w = \bar{w} + w'. \quad (3.1)$$

Here u, v, w are projections of hydrodynamic velocity vector at the axis x, y, z.

Further, Reynolds supposed that turbulent motions are periodical motions, that may be periodical either in space or in time. In accordance with these hypotheses on types of periodicity, averaging rules may be different.

I. *The averaging rules for motions periodic in space*

Let us separate out a physically small volume S. If u is the corresponding projection of velocity vector of any point inside S and ρ is liquid density, then upon summation over the volume S, we shall get the formula for determination of average velocity

$$\bar{u} = \frac{\sum\limits_{(S)} (\rho u)}{\sum\limits_{(S)} (\rho)}. \quad (3.2)$$

Similar formulae exist for projections $\bar{v}$ and $\bar{w}$.

Now in correspondence with formulae (3.1) velocities $\bar{u}, \bar{v}, \bar{w}$ may be taken for projections of the velocity vector of the centre of gravity of volume S. Velocities u', v', w' are the velocities of relative movement of liquid particles around the centre of gravity. It is obvious that at any moment of time these velocities will be functions of coordinates x, y, z. The centre of gravity of the separated volume travels in space. Let these movements be represented by $\Delta x, \Delta y, \Delta z_0$. Then the average velocity can be represented as follows

$$\bar{u} = \bar{u}_0 + \Delta x \left(\frac{\partial \bar{u}}{\partial x}\right)_0 + \Delta y \left(\frac{\partial \bar{u}}{\partial y}\right)_0 + \Delta z \left(\frac{\partial u}{\partial z}\right)_0$$
$$+ \frac{1}{2}(\Delta x)^2 \left(\frac{\partial^2 u}{\partial x^2}\right)_0 + \dots \quad (3.3)$$

Now we shall put formula (3.3) into equation (3.1), multiply both parts of the resulting equality by density ρ, sum up over volume S and, taking into account that

$$\sum\limits_{(S)} [\rho(u - \bar{u}_0)] = 0, \qquad \sum\limits_{(S)} [\rho(\Delta x)] = 0 \quad (3.4)$$

we shall get the formula for the average value of u' from

$$
\begin{aligned}
&\frac{\sum\limits_{(S)}(\rho u')}{\sum\limits_{(S)}(\rho)} \\
&= -\frac{1}{2}\left\{\left(\frac{\partial^2 u}{\partial x^2}\right)_0 \sum_{(S)}(\Delta x)^2 + \left(\frac{\partial^2 u}{\partial y^2}\right)_0 \sum_{(S)}(\Delta y)^2 \right. \\
&\qquad \left. + \left(\frac{\partial^2 u}{\partial z^2}\right)_0 \sum (\Delta z)^2\right\} + , \ldots \qquad (3.5)
\end{aligned}
$$

If we assume that inside of separated volume S

$$
\left(\frac{\partial u}{\partial x}\right)_0 = \left(\frac{\partial u}{\partial y}\right)_0 = \left(\frac{\partial u}{\partial z}\right)_0 = \text{const.} \quad (3.6)
$$

then we get

$$
\sum_{(S)}(\rho u') = 0.
$$

Similar considerations for $\bar{v}, \bar{w}$ give finally

$$
\sum_{(S)}(\rho u') = \sum_{(S)}(\rho v') = \sum_{(S)}(\rho w') = 0. \quad (3.7)
$$

II. *Averaging rules for motions periodic in time*

If we choose a time interval τ which would include time T, and carry out summation for this interval, we shall have for the average velocity the following formula:

$$
\bar{u} = \frac{\sum\limits_{(t)}(\rho u)}{\sum\limits_{(t)}(\rho)}. \quad (3.8)
$$

For any other moment of time t' we shall get

$$
\begin{aligned}
\bar{u} = \bar{u}_t &+ (t - t')\left(\frac{d\bar{u}}{dt}\right)_t + \frac{1}{2}(t - t')^2 \\
&+ \left(\frac{d^2 u}{dt^2}\right)_t + \ldots \qquad (3.9)
\end{aligned}
$$

Taking into account that

$$
\sum_{(t)}[\rho(t - t')] = 0, \qquad \sum_{(t)}[\rho(\bar{u}_t - u)] = 0
$$

we have

$$
\begin{aligned}
\frac{\sum\limits_{(t)}(\rho u')}{\sum\limits_{(t)}(\rho)} = &-\frac{1}{2}\sum_{(t)}(t - t')^2 \\
&\times \left(\frac{d^2\bar{u}}{dt^2}\right)_t - \ldots \qquad (3.10)
\end{aligned}
$$

Let us assume that within a singled out interval τ

$$
\left(\frac{d\bar{u}}{dt}\right)_t = \left(\frac{d\bar{v}}{dt}\right)_t = \left(\frac{d\bar{w}}{dt}\right)_t = \text{const.} \quad (3.11)
$$

Then we have

$$
\sum_{(t)}(\rho u') = \sum_{(t)}(\rho v') = \sum_{(t)}(\rho w') = 0. \quad (3.12)
$$

Formulae (3.7) and (3.12) allow formulation of the universal averaging rules which are correct both for the periodic movements with respect to space and for the periodic movements with respect to time:

$$
\begin{aligned}
&\bar{\bar{u}} = \bar{u}, \bar{u}' = 0, \bar{\bar{v}} = \bar{v}, \bar{v}' = 0 \\
&\bar{\bar{w}} = \bar{w}, \bar{w}' = 0.
\end{aligned} \qquad (3.13)
$$

Here barred quantities mean averaging operation either with respect to space or with respect to time.

This universality of formulae (3.13) allowed Reynolds to make no distinction between time and space periodicity but to call turbulent motions merely periodic motions.

In the same work published in 1894 Reynolds conducted averaging of the Navier–Stokes equations following rules (3.13). Thus he included constant velocity gradients within a physically small volume and for a physically short time into turbulent motion equations.

Reynolds' considerations discussed above probably served for Predvoditelev as the basis in his attempt to describe turbulent flows in terms of hydrodynamic equations corrected for nonideal continuity.

Moreover, Reynolds' averaging rules allow introduction of constant velocity gradients into hydrodynamics equations within a physically

small volume and Predvoditelev's hypothesis (2.7) includes any gradients.

In his work "On Turbulent Flows" prepared for publication as far back as in 1950* Predvoditelev suggested a hypothesis that turbulent flow appears because of the existence of a special kind of a discontinuous process due to compressibility of any liquid or gas.

This approach to a turbulent flow reduces to finding such a discontinuity front which would allow a fluid flow to possess all properties characteristic of turbulent flows.

To solve this problem, Predvoditelev made use of hydrodynamic equations such as (2.8) and Hugoniot–Hadamard method.

In case of monoatomic gas Predvoditelev's equation for a viscous vortex-free fluid flow is of the form

$$\rho \frac{dV}{dt} = - \text{grad } p + \mu \nabla^2 V + \lambda \text{ grad div } V$$

$$+ \rho\beta(\tfrac{1}{2} \text{ grad } V^2 + V \text{ div } V). \qquad (3.14)$$

It should be completed by the state equation in a general form

$$p = \phi(\rho, T). \qquad (3.15)$$

To describe the condition of heat input, we assume that expansion work is compensated by heat conduction and work of dissipative forces

$$p \text{ div } V = k\nabla^2 T - \mu(V \nabla^2 V)$$

$$- \lambda(\phi V \text{ grad div } V). \qquad (3.16)$$

Further Predvoditelev suggested that turbulent flow fluctuations are characterized by such a discontinuity front that first and second derivatives of the flow velocity, as well as first derivatives of pressure and density change sharply while passing this front. As concerns the temperature, it changes continuously together with all its derivatives.

In operation of passing through the front following Hugoniot–Hadamard's method in

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* This work of Predvoditelev will soon be published in *Problems of Physical Hydrodynamics*, ed. by A. V. Luikov

equations (3.14)–(3.16), Predvoditelev has arrived at the following formula for velocity

$$V^2 = \frac{p - \rho \, (\partial\phi/\partial\rho)}{\rho\beta}$$

$$\times \frac{1}{\{1 - [(1 - \beta)/\beta] \cos^2\theta\}} \qquad (3.17)$$

where θ is the angle between the velocity vector V and the normal to the front surface.

Formula (3.17) shows that at the wall where the discontinuity front touches it, the flow velocity V is different from zero but has a jump-wise change. Consequently, turbulent flow begins when the flow separates from the wall.

Near the wall no definite value can be ascribed to β as determination of flow dimensions at the wall is impossible. Therefore the expressions

$$\frac{p - \rho \, (\partial\phi/\partial\rho)}{\rho\beta} \quad \text{and} \quad \frac{1 - \beta}{\beta}$$

must be sought from experiment, thus it is convenient to express them in terms of velocity at the wall V_0 and at the axis V_m.

Now Predvoditelev has written formula (3.17) as

$$\phi = \frac{\phi_0}{\sqrt{(1 - (1 - \phi_0^2) \cos^2 \theta)}} \qquad (3.18)$$

where

$$\phi = \frac{V}{V_m}, \qquad \phi_0 = \frac{V}{V_0}.$$

Having defined the equation of the discontinuity front surface as a family of ellipses, Predvoditelev has presented the following formula for the angle θ

$$\cos \theta = \frac{(1 - y^2)}{1 - e^2 y^2}.$$

Here e is the eccentricity of the ellipse and y is the ratio of a local radius to the tube radius.

Thus formula (3.18) becomes

$$\phi = \frac{\phi_0}{\sqrt{\left[1 - (1 - \phi_0^2)\dfrac{(1 - y^2)}{(1 - e^2 y^2)}\right]}}. \quad (3.19)$$

Comparison of the above formula with experimental data of Nikuradse made by Prevoditelev has demonstrated that the formula gives a correct description of liquid flow in circular tubes.

In modern science on turbulent motions Prandtl's hypothesis on a laminar sublayer is is adopted. Later a concept of a transition layer that isolates laminar layer from a turbulent core was introduced.

Karman adopted this three-region model and used it as the basis for his formula of velocity distribution in turbulent flows which was considered for many years to be very useful and to agree fairly well with Nikuradse's experiments.

However, in 1949 the American worker Miller reported the work "Hypothesis of a Laminar Film". Here the author, using experimental data of Nikuradse convincingly proved that the latter matched his results so that they would agree with the above hypothesis.

Miller found that in Nikuradse's paper there was a disagreement between Table 2 that furnished results of direct measurements and Table 3 reporting experimental data expressed in a dimensionless form that was necessary for comparison with Karman's hypothesis.

In December of 1946 a discussion was held between Rose, Bethler and Professor Martinelli. Rose suggested to approach Prandtl, who directed the work of Nikuradse, concerning the above disagreement.

Prandtl replied that Nikuradse compiled a number of tables and plotted $\phi = V/V$ vs.

$\eta = vy/v$. He labelled this curve "old". Then Nikuradse plotted the straight line $\phi = \eta$ and labelled it "laminar". This line appeared to cross the "old" curve. That meant that some measured values appeared higher than they would be if the flow were not turbulent but laminar. As it was not acceptable, Nikuradse supposed that his measurements were not as accurate as he thought. If a laminar layer exists then the curve which corresponds to a turbulent region of the flow should be tangent to the straight line for a laminar flow at the boundary of the layer.

To obtain a curve for a turbulent flow region tangential to the straight line, Nikuradse added seven units to each η value, thus every point being displaced to the right by seven units.

But this displacement was merely a distortion of the data to make them compatible with the hypothesis.

This fact seems to be rather an evidence that no Prandtl laminar sublayer exists but there is velocity discontinuity at the wall.

Thus Predvoditelev's views on the nature of turbulent motions stated above do not disagree with experimental data and are waiting for further investigation.

REFERENCES

1. A. S. PREDVODITELEV, On molecular-kinetic basis of hydrodynamic equation, *Izv. Akad. Nauk SSSR. Otd. Tekhn. Nauk*, No. 4 (1948).
2. A. S. PREDVODITELEV, On rarefied gas aerodynamics and heat transfer problems, *Heat and Mass Transfer*, vol. 3, edited by A. V. LUIKOV and B. M. SMOLSKY. Energiya (1963).
3. A. S. PREDVODITELEV, Theory of heat and Riemannian manifolds, *Heat Transfer-Sov. Res.* 3, No. 1, 110 (1971).
4. *Osborne Reynolds and Engineering Science Today*, edited by D. M. McDOWELL and I. D. JACKSON. Barnes and Noble (1970).
5. W. MILLER, Hypothesis on laminar film, *Trans. ASME* No. 4 (1949).