**Austria, Member of ICHMT, EUROTHERM (1)**

**Centennial of Schrödinger Equation**
**Stephan Sponar**

## Centennial of Schrödinger Equation

**Privatdoz. Dipl.-Ing. Dr.techn. Stephan Sponar**
Atominstitut, TU Wien, Vienna, Austria
stephan.sponar@tuwien.ac.at

Erwin Schrödinger's famous, beautiful and well-known wave equation is, without any doubt, at the very hard of modern quantum mechanics and has revolutionized the understanding of nature for generations of physicists and future physicists. It was exactly 100 years ago that Schrödinger submitted his seminal paper "Quantisierung als Eigenwertproblem (Erste Mitteilung)" ("Quantisation as an Eigenvalue Problem, Part I") to "Annalen der Physik" on January 27 th in the year 1926 – the rest is history [1].

Villa Dr. Herwig in Arosa, Switzerland

The location of this epochal discovery was Arosa, an Alpine "Kurort" in Switzerland, located at an altitude at of roughly 1800 meter over sea, close to the great peak of the Weisshorn. In this beautiful location Erwin Schrödinger spent his Christmas holidays 1925/26. Instead of skiing *Erwin Schrödinger* there laid the foundation of his so-called wave-mechanics, described by the (time dependent) Schrödinger equation, named after him, denoted as

$$\mathrm{i}\hbar\frac{\partial}{\partial t}\psi(\vec{r},t) = \left(-\frac{\hbar^2}{2m}\Delta + V(\vec{r},t)\right)\psi(\vec{r},t) = H\psi(\vec{r},t)$$

Here the operator $H$ denotes the Hamiltonian of the system, $\hbar$ is the reduced Plank constant, $m$ is the mass of the particle. The solution $\psi(r, t)$ is referred to as wave function, containing all physical information of the system under investigation. The physical interpretation of the wave function, as





expressed by Max Born, is the following: "the probability of finding a particle in a certain region of space (a.k.a. interval) is given by the wavefunction's squared modulus within this interval". The question whether the wave function itself is a real physical object or just a mathematical tool to calculate probabilities is a subject of heated debates up to date [2]. Initial point of his wave approach was Schrödinger's dissatisfaction with the, from his point of view, unfounded, quantum condition in Niels Bohr's orbit theory [3].

Schrödinger's wave mechanics provided a new way to describe physical systems at the atomic scale, for instance a Hydrogen atom, by identifying the allowed energies as eigenvalues of the Schrödinger equation. Schrödinger's work paved the way for a wide range of modern technologies, ranging from semiconductors and lasers to emerging quantum devices and quantum sensors, used in quantum information processing and quantum communication.

Erwin Schrödinger 1933

There is a close relation between Schrödinger's wave-mechanic and Heisenberg's matrix mechanics developed by Werner Heisenberg, Max Born, and Pascual Jordan in 1925 [4]. Heisenberg's theory was actually the first complete physical description of an arbitrary quantum system. Heisenberg's matrix mechanics describes the temporal change, or time evolution, of matrices accounting for the system's observables (physical quantities of interest). Therefore, Heisenberg's Matrix mechanics is significantly closer to classical mechanics compared to Erwin Schrödinger's wave mechanics, which is a more abstract but compact description that has nevertheless prevailed. In Schrödinger's wave mechanics, time evolution is governed by the wave function or the state vector of the system; the observables (e.g. position and momentum) are time-independent. Only through the measurement process – the interaction of the wave function and the observable –the actual measured value determined. In other words; during the quantum measurement process "reality is created", in the sense of the Copenhagen school of Niels Bohr, Werner Heisenberg and Wolfgang Pauli. Not much later Schrödinger was also able to mathematically prove the full equivalence between his wave mechanics and Heisenberg's matrix mechanics, demonstrating that two seemingly different formalisms described the very same underlying quantum physic.

Erwin Schrödinger was born on August 12, 1887, in Vienna, as the only child of Rudolf Schrödinger a botanist, and Georgine Emilia Brenda Bauer, the daughter of a chemistry professor at TU Wien. His wide spectrum of different interests dated already from his high school years ranging from scientific disciplines, where he participated in various coemptions, to the logic of ancient grammar and poetry.

After finishing high school Schrödinger became a student at the University of Vienna from 1906 to 1910. During that time, he came under the strong influence of Fritz Hasenöhrl, who was Ludwig Boltzmann's successor, and developed interest in eigenvalue problems in the physics of continuous media, laying the foundation for his ground breaking theory years later. During the World War I Erwin Schrödinger served as a commissioned officer in the Austrian fortress artillery. In 1921, Schrödinger moved to Zurich, Switzerland where he was offered an academic position as assistant of Max Wien. This position was followed by employments at Stuttgart as an extraordinary professor, Breslau as ordinary professor, and then again at the University of Zurich replacing Max von Laue. It was during this period in Schrödinger's scientific career when he made his great discovery the wave mechanic, for which he was awarded the Nobel Prize for physics in 1933, shared with Paul A.M. Dirac.


Annemarie and Erwin Schrödinger's gravesite (Alpbach, Tyrol Austria) including Schrödinger equation

In 1927 Schrödinger succeeded Max Planck as Chair of Theoretical Physics at the University of Berlin. He held this position until the Nazi seizure of power in 1933, when he resigned his professorship in protest and emigrated to England, where he worked at Magdalen College, Oxford. However, in 1936 he returned to Austria and accepted a professorship at the University of Graz and in parallel became an honorary professor at the University of Vienna in the field of theoretical physics, until the annexation of Austria by the "German Reich" in 1938 forced him to flee again. First Schrödinger fled to Belgium in September 1938 where he was offered a visiting professorship at the University of Ghent. Only one later, in 1939, he was appointed by the then Prime Minister of Ireland, Éamon de Valera, to the newly founded Institute for Advanced Studies in Dublin, where he remained till the end of his active scientific career. After his retirement in 1956 Schrödinger returned to the now neutral Austria University of Ghent to become a "professor emeritus" at the University of Vienna. He died on the 4th of January, 1961 after a long illness.

**References**

[1] Schrödinger E, Quantization as a problem of proper values (Part IV), in Collected Papers on Wave Mechanics, New York, Providence, Rhode Island: AMS Chelsea Publishing, 2010, (Original: Annalen der Physik (4), vol. 81, pp. 109–139, 1926 [in German]), pp. 102–123 https://onlinelibrary.wiley.com/doi/10.1002/andp.19263861802





[2] R. Kastner, The Transactional Interpretation of Quantum Mechanics (Cambridge University Press, 2012).
[3] Bohr, N. "Atomic Theory and Mechanics1". Nature. 116 (2927): 845–852 (December 1925). Bibcode:1925Natur.116..845B. doi:10.1038/116845a0. ISSN 0028-0836
[4] W. Heisenberg, Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen, Zeitschrift für Physik, 33, 879-893, 1925 (received July 29, 1925). [English translation in: B. L. van der Waerden, editor, Sources of Quantum Mechanics (Dover Publications, 1968) ISBN 0-486-61881-1 (English title: "Quantum-Theoretical Re-interpretation of Kinematic and Mechanical Relations")


Important locations of Schrödinger's trajectory: from 1887 (Vienna) to 1927 (Zurich)


Important locations of Schrödinger's trajectory: from 1927 (Berlin) to 1961 (Vienna)